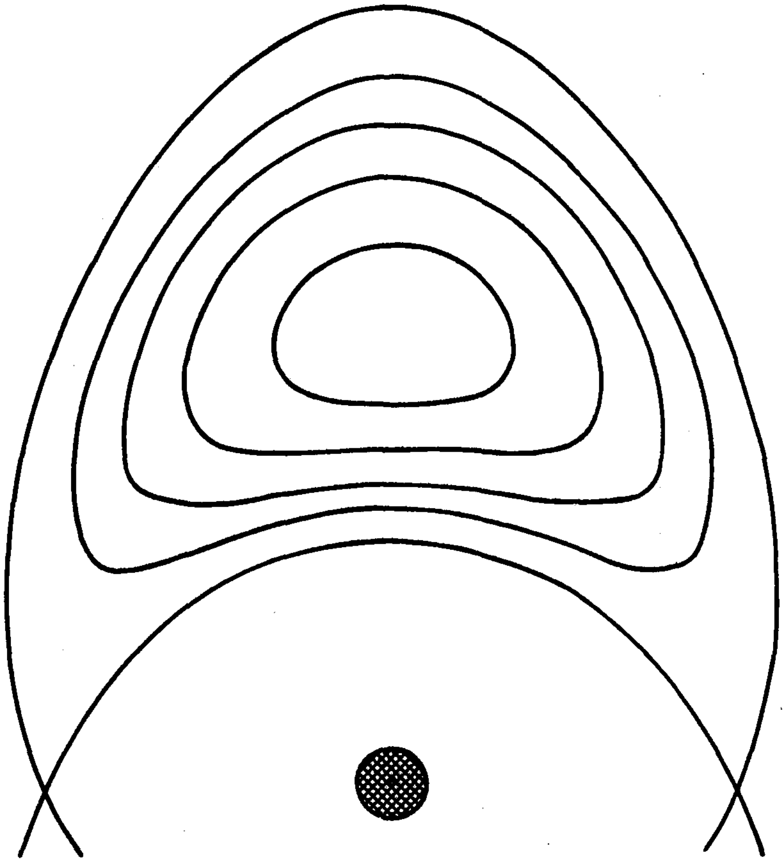
annual
SHERWOOD THEORETICAL MEETING

april 21-22, 1966



GENERAL DYNAMICS
General Atomic Division
JOHN JAY HOPKINS LABORATORY FOR PURE AND APPLIED SCIENCE
SAN DIEGO, CALIFORNIA

ANNUAL SHERWOOD THEORETICAL MEETING

General Information

1. All sessions will be held in Room T-217, Library Building, General Atomic.

2. Registration in Room T-214, will precede Session A.

3. Post-deadline papers will be scheduled as time permits after each session except Session A. Authors of post-deadline papers (defined as those whose abstracts do not appear in the program) should contact Nick Krall, the program chairman, as soon as possible for scheduling.

4. General Atomic will host a cocktail party to be held Thursday, 6:00pm at the Del Charro Penthouse. Transportation will be provided.

5. The research staff of the General Atomic Controlled Thermonuclear Research project would like to make your visit a pleasant one. If you need assistance please feel free to contact Norman Rostoker, Al Simon, Nick Krall, Ken Fowler, Don Pearlstein, Dave Chang or Tom O'Neil.

PROGRAM

Session A

Thursday Morning at 10:00
(N. Rostoker, presiding)

	E.C. Creutz	Introductory Remarks
A. 1	H. Grad	Guiding Center Equilibria
A. 2	E.T. Karlson	Stationary Equilibria of Toroidal Plasma
A. 3	T.G. Northrop	The Second Adiabatic Invariant and Guiding Center Motion in Ioffe Geometry
A. 4	H. Weitzner	Two Magnetohydrodynamics Problems and Ion Acoustic Waves
A. 5	W.H. Bostick D.R. Wells	Analysis of Pair Production of Plasma Vortices I
A. 6	D.R. Wells W.H. Bostick	Analysis of Pair Production of Plasma Vortices II
A. 7	W.B. Thompson	Unperformed Experiments on the interaction of radiation and a Plasma
A. 8	O. Buneman	Transconductance of a Stratified Plasma

Session B

Thursday Afternoon at 2:00
(T. K. Fowler, presiding)

B.1	H. Grad	Minimum-B as a Plasma Stability Criterion
B.2	A. Kadish	Linear Stability of Nonuniform Guiding Center Plasma
B.3	G.E. Guest C.O. Beasley, Jr.	Cold Plasma Effects in Finite Length Plasmas
B.4	P.H. Rutherford E.A. Frieman	On the Stabilization of Drift Waves by Shear
B.5	T. Kammash W.M. Farr	Ion Cyclotron Drift Instability in a Plasma with Cold Ion Species
B.6	R.W. Landau	Negative Mass Instability with B_θ Field
B.7	J. Lewak	Propagation of an Initial Disturbance in a Cold Plasma

Session C

Friday Morning at 9:30
(A. Simon, presiding)

C.1	E.G. Harris	Theoretical Interpretation of Some DCX-2 Observations. I.
C.2	R.A. Dory G.E. Guest	Theoretical Interpretation of Some DCX-2 Observations. II.
C.3	C.G. Smith A.S. Bishop	Effect of the Radial Electric Field on Classical Confinement in the Stellarator
C.4	W. Grossmann	Particle Loss in a Three-Dimensional Cusp
C.5	K. Hain	Two Dimensional θ-Pinch Calculations
C.6	F.C. Hoh	Onset of Turbulence in the Positive Column
C.7	L.D. Pearlstein	Low Frequency Instability of a Partially Ionized Plasma
C.8	D.E. Baldwin I.B. Bernstein	Structure of Cyclotron Harmonic Resonances in a Positive Column

Session D

Friday Afternoon at 2:00
(L. D. Pearlstein, presiding)

D. 1	G.K. Morikawa	On Steady Nonlinear Waves in a Warm Collision-free Plasma
D. 2	D. Montgomery	Parametric Resonance in Plasmas
D. 3	T.H. Dupree	Particle-Wave Interaction in Turbulent Plasma
D. 4	N.A. Krall	Heating Effects of a Universal Instability
D. 5	I.B. Bernstein J. Ahearne	Some Remarks on the Kinetic Theory of Plasmas
D. 6	J. Dawson	Ensembles of Vlasov Plasmas
D. 7	G. Rowlands D. Baldwin	Collective Damping of the Bernstein Modes
D. 8	A. Bers F.C. Hoh	Mechanism of Energy Transfer from Resonant Particles to Magnetic Field Perturbations

SESSION A

Thursday Morning at 10:00

GUIDING CENTER EQUILIBRIA

Harold Grad
Courant Institute of Mathematical Sciences
New York University

We have previously described local (or micro) instability in the GCF (guiding center fluid) and GCP (guiding center plasma) as nonposed initial value problems for the dynamical equations of motion. The problem of static equilibrium also yields well-posedness criteria which we again interpret in terms of stability. The GC equilibrium equations are the same whether the equations of motions are macroscopic (GCF) or involve a distribution function (GCP). The equilibrium momentum balance is incomplete. Depending on how this system is completed, we obtain the same micro-stability criteria as the dynamical GCF or the dynamical GCP. In the two-fluid case additional (independent) micro-stability criteria arise when the integral equation for the neutralizing potential has no solutions.

When the posedness conditions are satisfied, a general theory of open-ended GC equilibrium (and some toroidal theory) can be given, qualitatively similar to the scalar pressure case.

STATIONARY EQUILIBRIA OF TOROIDAL PLASMA

Erik T. Karlson[†]

Plasma Physics Laboratory
Princeton University

The stationary equilibria of a low-density plasma in toroidal geometry are studied theoretically. The magnetic surfaces are assumed to be concentric, circular toroids. It is assumed that plasma is injected with a source density rate Q, and a velocity $\bar{u}$ which may or may not be equal to the local plasma velocity. The effects of finite resistivity, inertia, and viscosity are taken into account. It is shown that if the resistivity term is small, there is a slowly diffusing equilibrium solution, with the plasma velocity uniquely determined by the injection velocity. The diffusion rate is the same as found by Pfirsch and Schlüter. It is shown that if the injection velocity is equal to the local plasma velocity the only possible motion is with a constant velocity parallel to the magnetic axis.

[†] On leave from A. B. Atomenergi, Stockholm, Sweden

THE SECOND ADIABATIC INVARIANT AND GUIDING CENTER MOTION IN IOFFE GEOMETRY

T. G. Northrop

Goddard Space Flight Center

The first correction [1] to the second adiabatic invariant explains numerical results obtained by SIAMBIS and TRIVELPIECE [2] for guiding-center trajectories in Ioffe mirror geometry. These numerical calculations show the guiding center oscillating about the surface defined by the lowest order term of the second invariant. The changes in amplitude and phase of the oscillations as the guiding center drifts are predicted by the first correction term.

$$J_0 = \oint (2m)^{1/2} [\overset{\frac{1}{2}mv^2}{K} - \mu_0 B(\alpha,\beta,s)]^{1/2} ds \qquad \frac{d}{dt}(\mu_0 + \epsilon\mu_1) = 0$$

$$\frac{1}{m}\frac{dJ_0}{dt} = \frac{\dot\alpha}{m}\frac{\partial J_0}{\partial \alpha} + \frac{\dot\beta}{m}\frac{\partial J_0}{\partial \beta} + \frac{\dot\mu_0}{m}\frac{\partial J_0}{\partial \mu_0} = \frac{d}{dt}\frac{T}{\epsilon}\int_0^t [\dot\beta\langle\dot\alpha\rangle - \dot\alpha\langle\dot\beta\rangle]$$

$$- \epsilon\dot\mu_1 \frac{\partial}{\partial \mu_0}\frac{J_0}{m}$$

α, β - see Kruskal

- Euler

[illegible] variables

$$\frac{\partial J_0}{\partial \alpha} = -eT\langle\dot\beta\rangle$$

$$\frac{\partial J_{..}}{\partial \beta} = eT\langle\dot\alpha\rangle$$

$$\cancel{\langle\dot\alpha\rangle}$$

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[1] T. G. Northrop, C. S. Liu, and M. D. Kruskal, Phys. Fluids, to be published.

[2] J. Siambis and A. W. Trivelpiece, Phys. Fluids, 8, 2047 (1965).

TWO MAGNETOHYDRODYNAMICS PROBLEMS AND ION ACOUSTIC WAVES

Harold Weitzner

Courant Institute of Mathematical Sciences
New York University

The slow expansion of a conducting piston into a plasma described by the linearized Lundquist equations is examined. A conical flow solution is obtained in special cases as a superposition of "subsonic" and "supersonic" flows matched at the piston and at a "transonic" curve. Without any "supersonic" flow, a solvable free boundary problem for the shape of the piston results. Some implications for the nonlinear expansion are given (work done jointly with Mr. Martin Snyder).

The slow steady flow of a plasma described by the linearized Lundquist equations past a non-conducting cylinder is considered. After certain shifts of the characteristics, necessary to ensure the existence of a physically reasonable solution, a solution is again obtained as a superposition of "subsonic" and "supersonic" flows. Implications for the non-linear case are discussed.

The plane wave oscillation problem is examined for electrons and ions described by the linearized Vlasov equation with no magnetic fields coupled by Poisson's equation. Both an electron plasma oscillation and an ion wave--either an ion acoustic wave or an ion plasma oscillation--are observable. The parameter ranges in which they may be seen and the nature of their damping--or dispersion--are given (work done jointly with Dr. Donald Dobrott).

ANALYSIS OF PAIR PRODUCTION OF PLASMA VORTICES† I and II.

Winston H. Bostick
Stevens Institute of Technology

Daniel R. Wells
University of Miami

Colinear, force-free vortex structure is essentially predicted by the work of Woltjer, Kruskal, and Chandrasekhar[1] in their variational calculations of the most stable equilibrium states. In their work the energy integral is varied subject to a set of integrals constraining the flow. The resulting stable structures can be related to large amplitude Alfven waves where some of the fluid is trapped in a "convective crest" and carried parallel to the magnetic field with the Alfven speed. The vortex filament is $\perp$ to the background magnetic field B_o ($\perp$-to-B_o tourbillon). This type of vortex structure has been produced in the conical theta pinch[2, 3] and in pairs in the plasma coaxial accelerator[4] where there are large gradients in the plasma density and "free surfaces" between plasma and magnetic field.

A theoretical model can be constructed to show how a Rayleigh-Taylor instability with its accompany velocity shear will grow into a pair of plasma vortex filaments which are $\parallel$ to the background magnetic field B_o ($\parallel$-to-B_o tourbillons). The model predicts that these vortices also should show some type of quasi-force-free magnetic field. Measured rotational velocity profiles of these vortices show that they rotate like rigid bodies, that is, without shear. These $\parallel$-to-B_o tourbillons have been observed to occur in pairs in the plasma flow over a dipole[4].

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[1] L. Woltjer, N. Acad. Sci. 44, 9, (1958).
[2] W. H. Bostick and D. R. Wells, Phys. Fluids 6, 1325 (1963).
[3] R. L. Small et al, AIAA, paper No. 66-155 Monterey, California, March 1966.
[4] Pair Production of Plasma Vortices, W. H. Bostick et al, submitted to Physics of Fluids.

† The study was supported by the Air Force Office of Scientific Research Grant No. AF-AFOSR-465-65, the Air Force Cambridge Research Laboratories, Office of Aerospace Research, under contract No. AF19(628)-2398, and by National Aeronautics and Space Administration under grant No. NsG-596 at Stevens Institute of Technology and the Atomic Energy Commission Contract No. AT40-(1)-3293 and Air Force Office of Scientific Research Grant No. 8365 at the University of Miami.

UNPERFORMED EXPERIMENTS ON THE INTERACTION OF RADIATION AND A PLASMA

W. B. Thompson

University of California, San Diego

TRANSCONDUCTANCE OF A STRATIFIED PLASMA[†]

O. Buneman

Institute for Plasma Research
Stanford University

A plasma in which the equilibrium conditions (density, drifts, d-c magnetic field, etc.) depend upon one space-coordinate such as x or r will transport small undulatory perturbations across its stratifications along the (unperturbed) orbits. The "transconductance kernel" $K(x, x')$ gives the current perturbations j at x due to an impressed field perturbation E at x' :

$$j(x) = \int K(x, x')\, E(x')dx'$$

$K(x, x')$ can be evaluated by summing the contributions from all the orbits that connect x and x'. In the collisionless (Vlasov) limit this transconductance appears, at first, to be pure imaginary and suggests purely reactive plasma behavior. However, in the transition from discrete to continuous orbit distributions one is led to singular integrals which result in real parts of the transconductance. As in the Landau analysis of a uniform plasma, these real parts could damp out potential instabilities. Examples of transconductance kernels will be presented.

[†] Work done on NSF grant GK-625

SESSION B

Thursday Afternoon at 2:00

MINIMUM-B AS A PLASMA STABILITY CRITERION

Harold Grad
Courant Institute of Mathematical Sciences
New York University

We describe a class of toroidal scalar pressure plasma equilibria with concentric flux surfaces in which a magnetic field minimum can be continuously varied from the interior to the exterior of the plasma with no apparent alteration of its stability. The spatial disposition of $V''(\Psi)$, frequently interpreted as a stability criterion, is unchanged as the absolute minimum of B moves from inside to outside the plasma. This casts doubt on a widely held intuitive belief concerning the relation of minimum-B to plasma stability and shows that "minimum-B" and "minimum-average-B" are essentially independent concepts. A brief discussion is given of the stability of anisotropic toroidal equilibria. Virtually nothing is known except for (1) local (micro) stability and (2) closed line interchange instability.

LINEAR STABILITY OF NONUNIFORM GUIDING CENTER PLASMA

Abraham Kadish
Courant Institute of Mathematical Sciences
New York University

Past investigations of nonuniform plasma stability have characteristically been restricted to special directions of propagation, restrictive ranges of pressure ratios, or strong limitations on the nature of the nonuniformity [1]-[4] (e.g., a jump discontinuity in the equilibrium parameters). Without any of these restrictions, we have derived sufficient conditions for the exponential stability of nonuniform guiding center [5] plasmas with a unidirectional field. These conditions are local analogues of those conditions which were previously shown to be sufficient for the boundedness of solutions of the initial value problem for the uniform plasma. It has also been shown that for a certain class of perturbations, instabilities weaker than exponential ones are always present as a result of the presence of gradients in the equilibrium configurations.

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[1] A. B. Mikhailovskii, Soviet Physics--JETP 17, 1043 (1963).
[2] N. A. Krall and M. N. Rosenbluth Phys. Fluids 6, 254 (1963).
[3] A. A. Galeev, Soviet Physics--JETP 17, 1292 (1963).
[4] S. Chandrasekhar et al., Proc. Roy. Soc. (London) A245, 435 (1958)
[5] G. Hellwig, Z. Naturforschung 1, 508 (1958).

COLD PLASMA EFFECTS IN FINITE LENGTH PLASMA[†]

G. E. Guest and C. O. Beasley, Jr.
Oak Ridge National Laboratory

Criteria are derived for the relative cold plasma density necessary to stabilize the grad B drift waves in finite length energetic plasmas separated from grounded conducting end plates by the cold plasma. For dense ($\omega_{pi}^2 >> \omega_{ci}^2$) plasmas in which the precession frequency of the energetic particles is very much less than the ion gyrofrequency, we find

$$(N_{cold}/N_{hot})_{critical} \sim 2\ell^2 \left(\frac{\omega_g}{\omega_{ci}}\right) \frac{L_H L_c}{2R^2} \frac{m}{M},$$

where ω_g is the precession frequency and we have taken $k_\perp = \ell/R$. For low density hot ion plasmas, a two-dimensional stability diagram is given which predicts a high density stable regime for extremely small ratios of N_{cold}/N_{hot}. A tentative comparison is made with some observed thresholds.

[†] Research sponsored by the U. S. Atomic Energy Commission under contract with the Union Carbide Corporation.

B.4

ON THE STABILIZATION OF DRIFT WAVES BY SHEAR

P. H. Rutherford and E. A. Frieman
Princeton University

The stabilization of electrostatic drift (universal) instabilities in a low-β collision-free inhomogeneous plasma has been reconsidered. The analysis of the perturbations in terms of normal modes has been abandoned in favor of an analysis of local perturbations which are in fact wave-packets of normal modes. These are found to be more difficult than the normal modes to stabilize by shear. The criterion for stabilization is that the wave-packets convect and diffuse out of the region of local instability before they have significantly exponentiated. This criterion has been calculated and it is found that for most of the wave-packet a shear of order $(m/M)^{\frac{1}{2}}$ is required, though parts of the wave-packet require finite shear. ($10^3 - 10^4$ for stellarator) – can get shear of this order in levitron

neglects curvature

$$l \sim \frac{a}{L}$$

ION CYCLOTRON DRIFT INSTABILITY IN A PLASMA WITH COLD ION SPECIES[†]

T. Kammash and W. M. Farr
The University of Michigan

In a recent [1] study of the stability of ion cyclotron electrostatic oscillations in a homogeneous plasma, it was found that cold ions destabilize a hot magnetized plasma and the instabilities occur at lower densities. In this investigation we examine the effect of cold ion species on the stability of ion cyclotron drift waves in a plasma with a density gradient in an arbitrary direction normal to the magnetic field. We find that the cold component has a similar destabilizing effect on these universal modes with a significant dependence on the angle between the propagation vector and the direction of the density gradient. Plots depicting the instability boundaries in terms of parameters containing the critical density will be shown and discussed.

[†] Work supported by the United States Atomic Energy Commission.

[1] L. S. Hall, W. Heckrotte and T. Kammash, Phys. Rev., 139, 4A, A1117 (1965).

NEGATIVE MASS INSTABILITY WITH B_θ FIELD

R. W. Landau

Physics International Company

We have extended the negative mass instability calculations of Nielsen[1] and Landau and Neil[2] to include the effects of a B_θ field, i.e., field lines through the torus whose radius is approximately that of the toroidal major radius. The torus is assumed to be radially thin. By linearizing and solving the kinetic equation, a new dispersion relation is obtained. We find that when $B_\theta \lesssim B_0$ (B_0 = mirror field), the growth rate and stability criteria obtained by Landau and Neil[2] are still valid. For $B_\theta > B_0$, stability may be obtained even by a spread in angular canonical momentum (or equivalent energy spread) too low to stabilize the beam under $B_\theta = 0$ conditions, if B_θ is large enough. These results appear to explain some recent results on the Stevens Institute plasma betatron [3]. In particular, the disruption of the beam appears to be due to this instability.

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[1] Nielsen, C. E., Sessler, A. M., and Symon, K. R., Proc. of International Conf. on Accelerators, CERN, 239 (1959).

[2] Landau. R. W., and Neil, V. K., UCRL-14406, University of Calif., Lawrence Radiation Laboratory, September 1965 (unpublished).

[3] Ferrari, L., and Rogers, K. C., Thesis, Stevens Institute of Technology, Hoboken, New Jersey, Figs. 23 and 25, May 1965.

SESSION C

Friday Morning at 9:30

THEORETICAL INTERPRETATION OF SOME DCX-2 OBSERVATIONS. I.†

E. G. Harris
The University of Tennessee
and
Oak Ridge National Laboratory

Some striking phenomena observed in DCX-2 are: (1) Ion cyclotron harmonics as high as the 100th have been observed. (2) Under some conditions only the even multiples of the ion cyclotron frequency appear. (3) A highly anisotropic plasma with $n \sim 10^8$ cm^{-3} and $T_\perp/T_\parallel \sim 10^3$ is sometimes built up and confined between shallow mirrors with a mirror ratio of about 1.001. (4) After the beam is turned off this highly anisotropic plasma does not emit the ion cyclotron harmonics. We shall discuss the interpretation of these observations in the light of theoretical knowledge of linear stability theory, quasi-linear theory, mode-mode coupling, finite plasma effects, and the effects of cold plasma. Part I will deal with the observations listed as (1) and (2).

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†Research sponsored by the U. S. Atomic Energy Commission under contract with the Union Carbide Corporation.

THEORETICAL INTERPRETATION OF SOME DCX-2 OBSERVATIONS. II.[†]

R. A. Dory and G. E. Guest
Oak Ridge National Laboratory

Some striking phenomena observed in DCX-2 are: (1) Ion cyclotron harmonics as high as the 100th. (2) Under some conditions only the even multiples of the ion cyclotron frequency appear. ~~(3) A highly anisotropic plasma with ion cyclotron frequency appear~~. (3) A highly anisotropic plasma with n $\sim 10^8$ cm^{-3} and $T_\perp/T_\parallel \sim 10^3$ is sometimes built up and confined between shallow mirrors with a mirror ratio of about 1.001. (4) After the beam is turned off this highly anistropic plasma is stable against the emission of ion cyclotron harmonics. We shall discuss the interpretation of these observations in the light of theoretical knowledge of linear stability theory, finite plasma effects, and the effects of cold plasma. Part I by Professor Harris dealt with the observations listed as (1) and (2). Part II is concerned with (3) and (4).

[†] Research sponsored by the U. S. Atomic Energy Commission under contract with the Union Carbide Corporation.

EFFECT OF THE RADIAL ELECTRIC FIELD ON CLASSICAL CONFINEMENT IN THE STELLERATOR.[†]

C. G. Smith and A. S. Bishop
Princeton University

Single-particle containment of a low density plasma in a stellarator is investigated by computing the guiding-center orbits of charged particles in the presence of a radial electric field. The preferential loss of ions from the velocity space "loss cone" results in the development of an electric field whose influence on particle motions is treated in a self-consistent manner. The introduction of this field tends to close the ion loss cone but, at the lower densities, not until a substantial number of particles have escaped. It tends to slightly open the electron loss cone in accord with the previously noted [1] competition between the electric field and the helical winding in determining the direction of rotation. The field also tends to produce a rather flat density profile.

(comment for author of C.2) in general should have machine length such that $\lambda_{ii} > L$

[†] Work performed under the auspicies of the U.S. Atomic Energy Commission.

[1] A. S. Bishop and C. G. Smith, "A Microscopic Treatment of Classical Containment in the Stellarator," Princeton Plasma Physics Lab. Rpt. MATT-403 (1966).

PARTICLE LOSS IN A THREE-DIMENSIONAL CUSP

William Grossmann
Courant Institute of Mathematical Sciences
New York University

In [1] the loss of particles through an axisymmetric cusped magnetic field containment geometry was considered. An appropriate adiabatic invariant was assumed for the plasma which was considered to be a high β, collisionless collection of particles contained within a field-free region bounded by a mathematically sharp interface. A velocity space loss cone criterion was derived and particle loss rates were calculated for a special form of the particle distribution function. Here, the correct form of the adiabatic invariant is derived using the method given by Gardner [2]. The particle loss rates using this invariant are compared with relevant experimental results [3]. Further, using the method of Gardner, a previous two-dimensional treatment of the present problem [4] is re-examined.

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[1] W. Grossmann, "Particle Loss in a Three-Dimensional Cusp," Paper Y10, APS-PPD Meeting, San Francisco, Nov. 8-11, 1965.

[2] C. S. Gardner, "Adiabatic Invariants of Periodic Classical Systems," Phys. Rev. 115 (1959), 791.

[3] E. M. Little et al., "Plasma End Losses and Heating in Low Pressure Regime of a Theta Pinch," Phys. Fluids 8 (1965, 1168.

[4] J. Killeen, "Theory of Cusped Geometries. IV. Particle Losses in Crossed Fields," Courant Inst. of Mathematical Sciences, New York Univ. Report NYO-9870, MF-8 (1960).

C.5

TWO DIMENSIONAL θ-PINCH CALCULATIONS

Klaus Hain

National Aeronautics and Space Administration
Goddard Space Flight Center

For many problems in magnetohydrodynamics inertia terms perpendicular to the fieldlines can be neglected, whereas parallel to the fieldlines the complete equations apply (i. e. in the later stages of the θ-pinch). A computing scheme has been developed for cylindrical symmetry. It is quasi stationary in r-direction. The fieldlines are the work unit~~e~~s in Z-direction. θ pinches with parallel fields can be treated. Preliminary results show the influence of mirror fields on the containment of the plasma.

C.6

ONSET OF TURBULENCE IN THE POSITIVE COLUMN

F. C. Hoh
Boeing Scientific Research Laboratories

A physical stabilization mechanism of the helical instability of the positive column due to quasi-linear effects is discussed. The quasi-linear steady state, having a single density and potential helix rotating around the axis, is perturbed. The resulting eigenvalue problem consists of a coupled chain of linearized differential equations. When the quasi-linear state next becomes unstable, a coupled set of helical modes, with different helicity and rotation velocity but with the same growth rate, is excited. These modes tend to lead the plasma into a turbulent state.

LOW FREQUENCY INSTABILITY OF A PARTIALLY IONIZED PLASMA

L. D. Pearlstein

General Atomic
Division of General Dynamics Corporation

In a recent paper[1] a low frequency instability of a weakly ionized plasma due to the unequal collision frequencies of ions and electrons with neutrals was reported. The result of this calculation showed instability developing independent of the sign of the equilibrium electric field in contrast with results previously obtained by A. Simon[2] and F. C. Hoh[3] in which the polarity of the electric field was all important. In his paper Morse[1] concluded that this added feature was due to the increased complexity of the density perturbations. The present calculation shows that the independence of stability upon the polarity of the electric field was strictly a consequence of an incorrect treatment of the streaming of electrons and ions to the end walls. Specifically, it is seen that the streaming to end walls adds a stabilizing contribution to the results of Simon[2].

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[1] D. L. Morse, Phys. Fluids **8**, 1339 (1965).

[2] A. Simon, Phys. Fluids **6**, 382 (1963).

[3] F. C. Hoh, Phys. Fluids **6**, 1184 (1963).

STRUCTURE OF CYCLOTRON HARMONIC RESONANCES IN A POSITIVE COLUMN

D. E. Baldwin and I. B. Bernstein
Yale University

Equations to describe the fine structure in the plasma resonance first observed by Buchsbaum and Hasegawa are derived from the Vlasov equation. The technique employed is to formally expand both the unperturbed equation and the linearized perturbed equation in inverse powers of the electron cyclotron frequency Ω, keeping the wave frequency ω to be of the same order as Ω. The expansion is carried out for arbitrary field and density variations, subject only to the restriction that the scale lengths be large compared with the electron Larmor radius. Motion along field lines is treated in the high phase velocity compared with thermal speed approximation. The general equations are specialized to a straight field and a cylindrically symmetric plasma. In the appropriate limit they reduce to a modified version of the equation treated by Buchsbaum and Hasegawa for the longitudinal modes and include the coupling to the transverse field. The phenomenon of the fine structure resonances is discussed in terms of these coupled equations.

SESSION D

Friday Afternoon at 2:00

D. 1

ON STEADY NONLINEAR WAVES IN A WARM COLLISION-FREE PLASMA

George K. Morikawa

Courant Institute of Mathematical Sciences
New York University

The speed and structure of weak nonlinear waves moving with uniform velocity in an oblique direction across a magnetic field in a collision-free plasma with temperature is obtained from a self-consistent formulation. The calculations describe how the speed and structure of these waves depend on the physical parameters--obliqueness angle, ion temperature, and electron temperature--for small electron to ion mass ratio. In contrast to the transverse magnetic field case, some care is necessary in handling those particles which 'turn around, "i. e., velocity reversal with respect to the unperturbed magnetic field direction.

Work with Gardner - Comm. NYU
w " Kleber

D. 2

PARAMETRIC RESONANCE IN PLASMAS

to be in Aug 71

David Montgomery

University of Iowa

Mechanical systems which break spontaneously into oscillation at Ω, a natural frequency, when some parameter of the system is forced to vary infinitesimally at frequency $\omega \approx 2\Omega$, are "parametrically resonant." Parametric resonance appears mathematically as exponentially growing solutions in a Krylov-Bogolyubov-Mitropolskii-Frieman perturbation theory, for solutions initially arbitrarily close to stationary equilibrium. From the higher orders in perturbation theory, a sequence of progressively weaker resonances appears at frequencies $\omega \approx 2\Omega, \Omega, 2\Omega/3 \ldots$. Silin[1] has shown the possibility of parametrically-resonant excitation of electrostatic waves in the cold electron-ion plasma. The exciting agency is an externally-imposed, spatially-uniform electric field. The small parameter of the problem may be taken to be the ratio of the electron excursion in the applied field to the wavelength of the parametrically excited wave. Silin's calculation has been generalized to avoid his purely electrostatic assumption, and to admit the possibility of transverse oscillations. Parametric excitation of transverse waves is also shown to be present, and often involves larger growth rates than for the electrostatic case. The escape from the plasma of transverse radiation may well prove to be the limiting factor in any attempted parametric excitation of a spectrum of electrostatic oscillations. This loss may be less important for the excitation of ionic sound waves, also demonstrable from a fluid model, with adiabatic equations of state for the electrons and protons. The characteristic frequencies in the ionic sound wave case will generally lie far below the electron plasma frequency, and thus not couple effectively to escaping transverse modes.

$\omega \approx 2\Omega$
times not
near Ω

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[1] V. P. Silin, Soviet Phys.--JETP 21, 1127 (1965).

a) electron osc.

in report ← b) electromagnetic waves $\left\{ \begin{array}{l} \omega_0 = \omega_p + \Omega_k \\ \Omega_k^2 = \omega_p^2 + c^2k^2 \end{array} \right.$

$\gamma_{max} \simeq a\omega_{pe}$

c) ionic sound

c) $\gamma_{max} \simeq a\omega_{pe}$

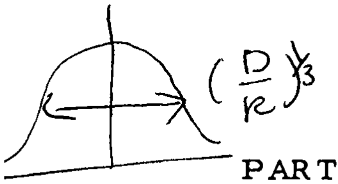
$\omega_0 \simeq 2\omega_s$

$$\left[\frac{\partial}{\partial t} + u\frac{\partial}{\partial x} - \frac{\partial}{\partial u} D(u)\frac{\partial}{\partial u}\right]\langle e^{-ikx^0(-t)}\rangle = 0$$

D.3

$$D(u) = \frac{q^2}{m^2}\sum |E_k|^2 \int_0^\infty d\tau\, e^{-i\omega\tau + ikx}\langle e^{-ikx(-t)}\rangle$$

put here



$$\langle e^{-ikx^0(-t)}\rangle \simeq e^{-ikx + ikut - k^2 D t^3}$$

PARTICLE-WAVE INTERACTION IN TURBULENT PLASMA[†]

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Utilizing a convergent perturbation theory of the Vlasov equation, the time evolution of the average distribution function ⟨f⟩ is studied for the case of strong turbulence. With certain approximations, ⟨f⟩ satisfies a velocity diffusion equation, with a diffusion coefficient determined by the relation

$$D(u) = \frac{\pi}{2u}\frac{q^2}{m^2} w^{-1}\int_{u-w}^{u+w} du' \left|E_{u'}\right|^2$$

where $w = \pi/2\,(\frac{D}{k})^{1/3}$ is the width of the wave-particle resonance. The electric field is written as a function of phase velocity u. This relation has two interesting limits. If the wave spectrum, $|E_u|^2$, is wide, then D approaches the usual quasi-linear result; if it is narrow, "trapping" occurs and D becomes proportional to the 3/2 power of the field, and the resonance width, w, becomes much larger than the spectrum width. The relation of ⟨f⟩ to wave growth and turbulent heating will be discussed.

not mode coupling

[†] This work was supported by the National Science Foundation (Grant GK-614).

$f = a_0 + a_1(E)E + a_2E^2 + \cdots$

$\Rightarrow$ exact particle trajectory leads to new resonance $\delta(kv + \omega)$

δ becomes wider

HEATING EFFECTS OF A UNIVERSAL INSTABILITY

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The quasi-linear theory of a low frequency electromagnetic universal instability[1] is developed. Electrostatic universal instabilities - modes driven by density gradients - have presented difficulties for the quasi-linear approach, due to the large growth rates ($\gamma \sim \omega$) and their necessary spatial dependence[2]. The electromagnetic mode examined here has smaller growth rates ($\gamma_{\max} << \omega$), is peaked in wavelength, and contains other features which make a quasi-linear attack seem attractive. Preliminary results indicate a transfer of energy from local pressure gradients into thermal energy, with the energy E_1^2 in the mode itself increasing more slowly. The original quasi-linear "flattening" of the distribution in this case appears to take place in coordinate space, at least locally, rather than in velocity space as in quasi-linear calculations of other modes[3].

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[1] N. A. Krall, M. N. Rosenbluth, Phys. Fluids 6, 254 (1963).

[2] N. A. Krall, M. N. Rosenbluth, Phys. Fluids 8, 1488 (1965).

[3] W. E. Drummond, D. Pines, Nuclear Fusion 1962 Supp. Part 3, 1049 (1962).

SOME REMARKS ON THE KINETIC THEORY OF PLASMAS

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It has been shown that the completely convergent kinetic equation for plasma electrons is just Boltzmann's equation with a cross section which depends on distribution function, and which for small angle scattering goes over into the Leonard Balescu form, while for large scale scattering the Rutherford cross section holds. It has also been shown that there is a qualitative difference in the space dependence of the two particle correlation function near thermal equilibrium from that far from thermal equilibrium. In the former case this leads to an effective two body interaction potential of exponential type; in the latter to one which falls off as the inverse cube of the interparticle separation.

ENSEMBLES OF VLASOV PLASMAS

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We consider an ensemble of plasmas; we assume that the plasma dynamics is accurately described by the Vlasov equation. The phase space point correlation functions are given by

$$F_s(\vec{r}_1, \vec{v}_1, \ldots, \vec{r}_s, \vec{v}_s) = \langle \prod_{i=1}^{s} f(r_i, v_i) \rangle_{av}$$

where f(r, v) is the distribution function for one Vlasov system and the average is taken over the ensemble. A hierarchy of equations for the F's can be obtained by taking averages of the Vlasov equation. The heirarchy would be identical to the BBGKY hierarchy for a plasma of discrete particles except for the fact that the terms due to the direct interactions between $\langle s \rangle$ particles are absent. Since for unstable and turbulent plasmas these terms would generally be small, this suggests that as far as such plasmas are concerned most correlation effects are correctly given by the Vlasov equation. One can solve this hierarchy by methods similar to those employed for the BBGKY hierarchy. By assuming two phase point correlations are small and that three point correlations are negligible, one obtains the equations of Quasi Linear theory. Other methods of solution and various aspects of them will be discussed.

COLLECTIVE DAMPING OF THE BERNSTEIN MODES

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The propagation of electrostatic waves perpendicular to a uniform magnetic field, the so-called Bernstein modes, are discussed. It is shown in the limit as the strength of the magnetic field approaches zero, that these modes, which themselves are undamped, collectively act so as to form a single damped mode identical to the usual Landau damped mode appropriate to zero magnetic field. The behaviour of the plasma for small but finite magnetic field is also considered. It is shown that damping due to phase mixing exists for a time short compared to the reciprocal of the cyclotron frequency. In times long compared to this, the plasma exhibits a quasi-periodic behaviour.

MECHANISM OF ENERGY TRANSFER FROM RESONANT PARTICLES TO MAGNETIC FIELD PERTURBATIONS

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and

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The mechanism of energy transfer from resonant electrons to electrostatic waves, i.e., Landau growth, is well known in the case of Langmuir oscillations[1,2] as well as universal instabilities[3]. In this note we show how resonant particles can transfer energy to induced electric fields and hence cause amplification of magnetic field perturbations. This mechanism explains the instability of the sheet pinch[4,5].

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[4] G. Laval, R. Pellat, and M. Vuillemin, Culham Conf. Sept. 6-19, 1965.
[5] F. C. Hoh, Phys. Fluids 9, 277 (1966).

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NOTES

PROPAGATION OF AN INITIAL DISTURBANCE IN A COLD PLASMA

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Propagation parallel to the magnetic field of an initial current pulse at the origin is studied using the two-fluid cold plasma equations and neglecting the displacement current. The asymptotic solutions show a wave front travelling with a characteristic velocity of $\frac{3\sqrt{3}}{8\gamma}(1 + \gamma^2) C_A$ (where γ^2 is the electron-ion mass ratio and C_A the Alfvén speed), plus a stationary (non-propagating) disturbance at the origin which oscillates at the sum of the gyro-frequencies and has a wave number which spreads with time like $t^{1/2}$.

CHANGES TO PROGRAM

SESSION B

B. 7 -- J. Lewak. "Propagation of an Initial Disturbance in a Cold Plasma."

SESSION C

C. 2--Delete in line 3, "(3) A highly anisotropic plasma with ion cyclotron frequency appear.", from printed abstract.

C. 6--Last line: for turbulend read turbulent.